Light Water Reactor Sustainability Program

Grizzly Usage and Theory Manual
Version 1.0 Beta

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March 2016

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Acknowledgments

Contents

List of Figures

1 Introduction

Grizzly is a multiphysics simulation code for characterizing the behavior of nuclear power plant (NPP) structures, systems and components (SSCs) subjected to a variety of age-related degradation mechanisms. Grizzly simulates both the progression of aging processes, as well as the capacity of aged components to safely perform.

This initial beta release of Grizzly includes capabilities for engineering-scale thermo-mechanical analysis of reactor pressure vessels (RPVs). Grizzly will ultimately include capabilities for a wide range of components and materials. Grizzly is in a state of constant development, and future releases will broaden the capabilities of this code for RPV analysis, as well as expand it to address degradation in other critical NPP components.

1.1 The role of Grizzly in multiscale modeling of RPVs

RPVs are put into service with large populations of flaws introduced in the manufacturing process. During normal or abnormal conditions, transients in the temperature and pressure of the coolant can induce elevated stresses, which results in stress concentrations at the tips of these pre-existing flaws. To determine whether these stress concentrations are below the threshold to initiate fractures at those flaws requires a detailed engineering analysis. As the RPV steel is exposed to irradiation and elevated temperature, the microstructure evolves and the steel becomes more brittle, making it more susceptible to fracture initiation and growth from flaws. This increased susceptibility to fracture is an important consideration when contemplating long term operation beyond 60 years.

A number of embrittlement trend curves have been developed to represent the effect of long-term environmental exposure and are currently employed in practice in engineering fracture assessments of RPVs. These are based on data extending to the lifetime of the current reactor fleet, and hence can not be confidently extrapolated to applications beyond 60 years of service. For more confidence in predictions of material behavior under long term operation, a science-based, bottom-up capability to simulate phenomena leading to embrittlement at lower length scales is necessary. This will permit evaluation of behavior outside the range of applicability of the physically motivated, empirically calibrated models currently used for that purpose.

The goal of the development effort for RPVs in Grizzly is to provide capabilities for both engineering-scale fracture mechanics and for modeling microstructure evolution, to provide a predictive simulation tool that can improve the confidence in predictions of the fracture response of very general flaw configurations, including the effects of microstructure evolution on the material's resistance to fracture after long-term exposure to irradiation and high temperatures.

Because of the multiple scales and physical phenomena involved, multiple modeling techniques are being developed for RPVs in Grizzly. A map of the methods employed is shown in Figure 1, demonstrating how the microstructure evolution and engineering scale models will be used together in a probabilistic risk assessment framework to compute a probability of failure of the RPV under off-normal conditions, accounting for the effects of age-related material degradation.

At the engineering scale, Grizzly first computes the thermo-mechanical response (stress, temperature) of RPVs under pressurized thermal shock scenarios. Local fracture submodels then use these results to compute the stress intensity at pre-existing flaws and determine whether fracture propagation will occur. These flaw region fracture models are used in a probabilistic simulation, which is difficult to do directly using the models because of the high computational costs involved. Reduced order models will be developed using these detailed fracture models to efficiently evaluate their response within a probabilistic model using the RAVEN code. Within the probabilistic evaluation, an embrittlement trend curve is employed to represent the effect of exposure to high neutron flux and temperature on the toughness of the material.

The RPV modeling roadmap shows two parallel paths to model the two dominant mechanisms leading

Grizzly RPV Multiscale Research Map
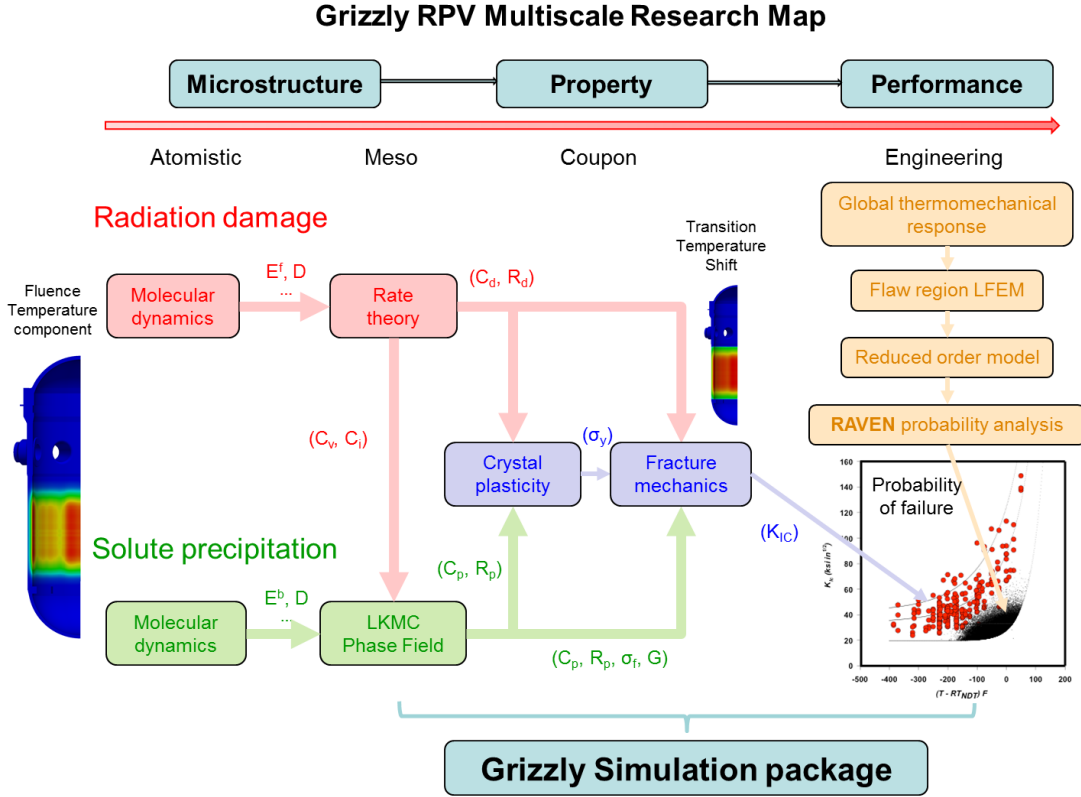
Figure 1: Map for models being developed for RPV embrittlement modeling with Grizzly

to RPV steel embrittlement: irradiation damage and solute precipitation. These mechanisms both lead to material hardening and embrittlement, which will be quantified using other models. The methods involved span multiple length and time scales, including molecular dynamics, lattice kinetic Monte Carlo (LKMC), rate theory (of which cluster dynamics is a subset), crystal plasticity and fracture mechanics. Some of these models (e.g. atomistic models) by necessity are developed using other codes, but they are being developed in Grizzly wherever applicable. These are still under development, but will be included in future releases as they are completed.

1.2 Software Environment

Grizzly is based on the MOOSE multiphysics simulation platform developed at Idaho National Laboratory (INL). MOOSE provides a general platform for solving a variety of physics problems, including LWR component aging problems. MOOSE provides capabilities for solving a general set of coupled partial differential equations using the finite element method. This includes support for equation solvers, file input/output, infrastructure for communication between processors in a parallel high performance computing environment, finite element data structures and functions, and a modular architecture that permits incorporation of physics models within an application derived on MOOSE, such as Grizzly.

Because of the highly modular nature of the MOOSE framework, it is easy to compose together physics modules from a variety of sources. MOOSE is distributed with both the core framework, as well as a basic set of physics modules, which provide capabilities for physics shared by multiple end applications.

Many of the capabilities of Grizzly are provided by the `solid_mechanics` and `heat_conduction` mod-

ules in the MOOSE code base. From the user's perspective, the method for incorporating a set of physics modules in an analysis is independent of where the source code for those modules exists. This document covers the usage of the set of capabilities needed to apply Grizzly to LWR aging problems of interest. Those capabilities that are general-purpose are implemented in MOOSE modules, while Grizzly is a repository for capabilities specific to LWR aging problems.

1.3 Document Organization

This document includes the following sections:

- Section 2 documents the procedures for obtaining and compiling the Grizzly source code, and for running Grizzly.

- Section 3 provides a summary of the structure of the Grizzly input file.

- Section 4 summarizes the input file syntax for features common to all types of physics models in MOOSE-based codes.

- Section 5 summarizes the governing equations and associated commands for thermal analysis.

- Section 6 summarizes the governing equation, kinematics, and associated commands for mechanical analysis.

- Section 7 summarizes the procedure and commands used for submodeling.

- Section 8 summarizes the input file syntax for reactor pressure vessel fracture analysis beyond what is covered in previous sections, with accompanying discussion of the theory behind the code features used.

- Section 9 shows full input files for global thermo-mechanical RPV analyses and fracture analyses of local flaws, with commentary on the features and options used.

2 Running Grizzly

2.1 Checking Out the Code

Grizzly is hosted on an internal GitLab server in INL's High Performance Computing enclave. The instructions for checking out the code are different depending upon whether you are an internal (INL onsite user) or external user. These instructions are only for checking out and running the code. If you plan to contribute to Grizzly, detailed instructions for contributing can be found on the idaholab/grizzly wiki page on GitLab.

2.1.1 Internal Users

The first step is to obtain an INL High Performance Computing (HPC) account. Once HPC access has been granted go to the GitLab website (https://hpcgitlab.inl.gov/) and login with your HPC username and password on the LDAP tab.

Once logged in and access has been granted to the idaholab/grizzly repository the following steps are required for the initial checkout of the code:

First add an SSH key to your GitLab profile. To do so execute the code below in a terminal.

```
ssh-keygen -t rsa -C "your_email"
cat ~/.ssh/id_rsa.pub
```

Then copy-paste the key to the 'My SSH Keys' section under the 'SSH' tab in your user profile on GitLab. Next clone the Grizzly GitLab repository.

```
cd ~/projects/
git clone git@hpcgitlab.inl.gov:idaholab/grizzly.git
```

Next initialize the MOOSE submodule:

```
cd ~/projects/grizzly/
git submodule update --init
```

It is necessary to build libMesh before building any application:

```
cd ~/projects/grizzly/moose/scripts
./update_and_rebuild_libmesh.sh
```

Once libMesh has compiled successfully, you may now compile Grizzly:

```
cd ~/projects/grizzly/
make (add -jn to run on multiple "n" processors)
```

Once Grizzly has compiled successfully, it is recommended to run the tests to make sure the version of the code you have is running correctly.

```
cd ~/projects/grizzly/
./run_test (add -jn to run "n" jobs at one time)
```

2.1.2 External Users

For external users there are a few additional steps to checking out the code. First request an HPC account. This can be requested at the HPC registration page (https://secure.inl.gov/caims/). Once an HPC account has been generated an ssh tunnel will need to be set up to access GitLab. Add the following lines to your ~/.ssh/config file. Replace <USERNAME> with the username for your HPC account.

```
#Multiplex connections for less RSA typing
Host *
    ControlMaster auto
    ControlPath ~/.ssh/master-%r@%h:%p

# General Purpose HPC Machines
Host eos hpcsc flogin1 flogin2 quark
    User <USERNAME>
    ProxyCommand ssh <USERNAME>@hpclogin.inl.gov netcat %h %p

#GitLab
Host hpcgitlab.inl.gov
    User <USERNAME>
    ProxyCommand nc -x localhost:5555 %h %p

#Forward license servers, webpages, and source control
Host hpclogin hpclogin.inl.gov
    User <USERNAME>
    HostName hpclogin.inl.gov
    LocalForward 8080 hpcweb:80
    LocalForward 4443 hpcsc:443
```

Next create a tunnel into the HPC environment and leave it running while you require access to GitLab. If you close this window, you close the connection:

```
ssh -D 5555 username@hpclogin.inl.gov
```

Then you have to adjust your socks proxy settings for your web browser to reflect the following settings

```
hostname: localhost
port: 5555
```

If you do not know how to do that, look up **Change socks proxy settings for <insert the name of your web browser here>** on google.com or some other search engine. Once that is complete you can login to the GitLab website. The rest of the steps for checking out the code are the same as for internal users.

2.2 Updating Grizzly

If it has been some time since you have checked out the code an update will be required to gain access to the new features within Grizzly. The following instructions apply to both internal and external users to update the code. Note that external users must have their ssh tunnel set up prior to proceeding. First update Grizzly:

```
cd ~/projects/grizzly/
git pull
```

Then update the MOOSE submodule:

```
cd ~/projects/grizzly/
git submodule update
```

Next rebuild libMesh:

```
cd ~/projects/grizzly/moose/scripts/
./update_and_rebuild_libmesh.sh
```

And finally recompile Grizzly:

```
cd ~/projects/grizzly/
make (add -jn to run on multiple "n" processors)
```

2.3 Executing Grizzly

When first starting out with Grizzly, it is recommended to start from an example problem similar to the problem that you are trying to solve. Multiple examples can be found at grizzly/examples/ and grizzly/assessment/. It may be worth running the example problems to see how the code works and modifying input parameters to see how the run time, results and convergence behavior change.

To demonstrate running Grizzly, consider the 2d strip model of the global response of an RPV included in the Grizzly distribution:

```
cd ~/projects/grizzly/demonstration/rpv_global/2d_strip
# To run with one processor
~/projects/grizzly/grizzly-opt -i 2d_strip.i
# To run in parallel (2 processors)
mpiexec -n 2 ../../grizzly-opt -i 2d_strip.i
```

Note that the procedure for running this model in parallel is shown only for illustrative purposes. This particular model is quite small, and would not benefit from being run in parallel, although it can be run that way.

2.4 Getting Started

2.4.1 Input to Grizzly

Grizzly simulation models are defined by the user through a text file that defines the parameters of the run. This text file specifies the set of code objects that are composed together to simulate a physical problem, and provides parameters that control how those objects behave and interact with each other. This text file can be prepared using any text editor.

In addition to the text file describing the model parameters, Grizzly also requires a definition of the finite element mesh on which the physics equations are solved. The mesh can be generated internally by Grizzly using parameters defined in Grizzly's input file for very simple geometries, or can be read from a file as defined in the Grizzly input file. Grizzly supports the full set of mesh file formats supported by MOOSE, although the most common mesh format is the ExodusII format.

2.4.2 Post Processing

Grizzly typically writes solution data to an ExodusII file. Data may also be written in other formats, a simple comma separated file giving global data being the most common.

Several options exist for viewing ExodusII results files. These include commercial as well as open-source tools. One good choice is Paraview, which is open-source.

Paraview is available on a variety of platforms. It is capable of displaying node and element data in several ways. It will also produce line plots of global data or data from a particular node or element. A complete description of Paraview is not possible here, but a quick overview of using Paraview with Grizzly results is available in the Grizzly workshop material.

2.4.3 Graphical User Interface

It is worth noting that a graphical user interface (GUI) exists for all MOOSE-based applications. This GUI is named Peacock. Information about Peacock and how to set it up for use may be found on the MOOSE wiki page.

Peacock may be used to generate a text input file. It is also capable of submitting the analysis. It also provides basic post processing capabilities.

3 Input File Structure

3.1 Basic Syntax

The input file used by Grizzly (or any MOOSE application) is broken into sections or blocks identified with square brackets. The type of input block is placed in the opening brackets, and empty brackets mark the end of the block.

```
[BlockName]
  <block lines and subblocks>
[]
```

Each block may contain an arbitrary number of line commands to define parameters related to that block. They can also optionally contain one or more subblocks, which may in turn contain their own nested sub-blocks. Subblocks are opened and closed with the following syntax:

```
  [./subblock_name]
    <line commands>
  [../]
```

Note that all subblocks contained within a given block must have unique names (within the opening [] brackets).

Parameters are defined with line commands, and are given as key/value pairs separated by an equals sign (=). They specify parameters to be used by the object being described. The key is a string (no whitespace), and the value may be a string, an integer, a real number, or a list of strings, integers, or real numbers. Lists are given in single quotes and are separated by whitespace.

The following are examples of line commands for a single parameter and for a list of parameters:

```
single_parameter = 2.3
list_of_parameters = '1.0 2.3 3.7'
```

Blocks and subblocks at any level can contain line commands, which must be appropriate for the scope of the block containing them. Most of the time, blocks are used to create instances of MOOSE objects, and contain a `type` = parameter to specify the type of MOOSE object to be created. The name of the MOOSE object specified in the parameter corresponds to the name of the class in the C++ source code.

Each object type has a unique set of input parameters that are valid for specifying the behavior of that object. Some parameters are required, and some are optional, and revert to default behavior if they are not specified. An error message is generated if a line command does not apply within the scope in which it is provided. Repeating a line within a block also results in an error.

In this document, line commands are shown with the keyword, an equal sign, and, in angle brackets, the value. If a default value exists for that line command, it is shown in parentheses.

In the initial description of a block, line commands common to all subblocks will be described. Those line commands are then omitted from the description of the subblocks but are nonetheless valid line commands for those subblocks.

The name of a subblock (`[./<name>]`) is arbitrary. However, these names should be chosen to be meaningful because they can be used to refer to those entities elsewhere in the input file. Not every created entity is referenced elsewhere, but a name must be created for every entity regardless.

3.2 Summary of MOOSE Object Types

MOOSE is an objected-oriented system with well-defined interfaces for applications to define their own physics-specific code modules. The following is a listing of the major types of MOOSE objects used by Grizzly:

- Variable

- Kernel

- AuxVariable

- AuxKernel

- Material

- BoundaryCondition

- Function

- Postprocessor

- VectorPostprocessor

- Constraint

- Damper

Specialized versions of these object types are implemented to provide the functionality needed to model physics of interest for Grizzly.

3.3 Grizzly Syntax Page

A complete listing of all input syntax options in MOOSE is available on the MOOSE Documentation page. See the section on Input File Documentation. Note also that you can run

```
grizzly-opt --dump
```

to get a list of valid input options for Grizzly.

3.4 Units

Grizzly can be run using any unit system preferred by the user. Empirical models within Grizzly that depend on a specific unit system are noted in this documentation.

4   Common Commands For All Physics

4.1   GlobalParams

The `GlobalParams` block specifies parameters that are available, as appropriate, in any other block or sub-block in the input file. For example, consider a subblock that accepts a line command with the keyword `value`. If the subblock has a line command for `value`, that line command will be used regardless of what is in `GlobalParams`. However, if the line command is missing in the subblock but defined in `GlobalParams`, the subblock will use the parameter defined in `GlobalParams`. In the example below, the line commands `order = FIRST` and `family = LAGRANGE` will be available in all other blocks and subblocks defined in the input file.

```
[GlobalParams]
  order = FIRST
  family = LAGRANGE
[]
```

4.2   Problem

The `Problem` block is optionally used to define parameters for the `Problem` object. It only needs to be included to specify non-default behavior. Typically, the only parameter set here is `coord_type`, which specifies that the model should be treated as axisymmetric (`RZ`) or spherically symmetric (`RSPHERICAL`). For 3D or 2D plane strain models, this block may be omitted.

```
[Problem]
  coord_type = <string>
[]
```

4.3   Mesh

The `Mesh` block's purpose is to give details about the finite element mesh to be used. Externally created meshes can be imported from separate files in a variety of formats supported by the libMesh library that is used by MOOSE for finite element functionality. The most commonly used mesh is ExodusII.

Typically meshes are created using the mesh generation tool Cubit (for U.S. government users) or Trelis (the commercialized version of Cubit).

```
[Mesh]
  file = <string>
  displacements = <string list>
  patch_size = <integer> (40)
[]
```

`file`	**Required.** The name of the file containing the mesh.
`displacements`	List of the displacement variables used to create the displaced mesh. This line must be given if the model is to use contact or large displacement theory. Typically `disp_x` `disp_y` `disp_z` for a 3D model.

4.4 Executioner

The `Executioner` block describes how the simulation will be executed. It includes commands to control the solver behavior and timestepping. Time stepping is controlled by a combination of commands in the `Executioner` block, and the `TimeStepper` block nested within the `Executioner` block.

```
[Executioner]
  type = <string>
  solve_type = <string>
  petsc_options = <string list>
  petsc_options_iname = <string list>
  petsc_options_value = <string list>
  line_search = <string>
  l_max_its = <integer>
  l_tol = <real>
  nl_max_its = <integer>
  nl_rel_tol = <real>
  nl_abs_tol = <real>
  start_time = <real>
  dt = <real>
  end_time = <real>
  num_steps = <integer>
  dtmax = <real>
  dtmin = <real>
  [TimeStepper]
    #TimeStepper commands
  [../]
```

type	**Required**. Several available. Typically `Transient`.
solve_type	One of `PJFNK` (preconditioned JFNK), `JFNK` (JFNK), `NEWTON` (Newton), or `FD` (Jacobian computed by finite difference–serial only, slow).
petsc_options	PETSc flags.
petsc_options_iname	Names of PETSc name/value pairs.
petsc_options_value	Values of PETSc name/value pairs.
line_search	Line search type. Typically none.
l_max_its	Maximum number of linear iterations per solve.
l_tol	Linear solve tolerance.
nl_max_its	Maximum number of nonlinear iterations per solve.
nl_rel_tol	Nonlinear relative tolerance.
nl_rel_abs	Nonlinear absolute tolerance.
start_time	The start time of the analysis.
end_time	The end time of the analysis.
num_steps	The maximum number of timesteps.
dtmax	The maximum allowed timestep size.
dtmin	The minimum allowed timestep size.

Several `Executioner` types exist, although the `Transient` type is typically the appropriate one to use for transient analyses.

Similarly, many PETSc options exist. Please see the online PETSc documentation for details.

4.5 Timestepping

The method used to calculate the size of the timesteps taken by MOOSE is controlled by the `TimeStepper` block. There are a number of types of `TimeStepper` available. Three of the types most commonly used are described here. These permit the timestep to be controlled directly by providing either a single fixed timestep to take throughout the analysis, by providing the timestep as a function of time, or by using adaptive timestepping algorithm can be used to modify the time step based on the difficulty of the iterative solution, as quantified by the numbers of linear and nonlinear iterations required to drive the residual below the tolerance required for convergence.

4.5.1 ConstantDT

The `ConstantDT` type of `TimeStepper` simply takes a constant timestep size throughout the analysis.

```
[TimeStepper]
  type = ConstantDT
  dt = <real>
[../]
```

type	ConstantDT
dt	**Required**. The initial timestep size.

`ConstantDT` begins the analysis taking the step specified by the user with the `dt` parameter. If the solver fails to obtain a converged solution for a given step, the executioner cuts back the step size and attempts to advance the time from the previous step using a smaller time step. The timestep is cut back by multiplying the timestep by 0.5.

If the solution with the cut-back timestep is still un-successful, it is repeatedly cut back until a successful solution is obtained. The user can specify a minimum timestep through the `dtmin` parameter in the `Executioner` block. If the timestep must be cut back below the minimum size without obtaining a solution, the code will exit with an error. If the timestep is cut back using `ConstantDT`, that cut-back step size will be used for the remainder of the the analysis.

4.5.2 FunctionDT

If the `FunctionDT` type of `TimeStepper` is used, time steps vary over time according to a user-defined function.

```
[TimeStepper]
  type = FunctionDT
  time_t = <real list>
  time_dt = <real list>
[../]
```

```
type        FunctionDT
time_t      The abscissas of a piecewise linear function for timestep size.
time_dt     The ordinates of a piecewise linear function for timestep size.
```

The timestep is controlled by a piecewise linear function defined using the `time_t` and `time_dt` parameters. A vector of timesteps is provided using the `time_dt` parameter. An accompanying vector of corresponding times is specified using the `time_t` parameter. These two vectors are used to form a timestep vs. time function. The timestep for a given step is computed by linearly interpolating between the pairs of values provided in the vectors.

The same procedure that is used with `ConstantDT` is used to cut back the timestep from the user-specified value if a failed solution occurs.

4.5.3 IterationAdaptiveDT

The `IterationAdaptiveDT` type of `TimeStepper` provides a means to adapt the timestep size based on the difficulty of the solution.

```
[TimeStepper]
  type = IterationAdaptiveDT
  dt = <real>
  optimal_iterations = <integer>
  iteration_window = <integer> (0.2*optimal_iterations)
  linear_iteration_ratio = <integer> (25)
  growth_factor = <real>
  cutback_factor = <real>
  timestep_limiting_function = <string>
  max_function_change = <real>
  force_step_every_function_point = <bool> (false)
[../]
```

`dt`	**Required**. The initial timestep size.
`optimal_iterations`	The target number of nonlinear iterations for adaptive timestepping.
`iteration_window`	The size of the nonlinear iteration window for adaptive timestepping.
`linear_iteration_ratio`	The ratio of linear to nonlinear iterations to determine target linear iterations and window for adaptive timestepping.
`growth_factor`	Factor by which timestep is grown if needed.
`cutback_factor`	Factor by which timestep is cut back if needed.
`timestep_limiting_function`	Function used to control the timestep.
`max_function_change`	Maximum change in the function over a timestep.
`force_step_every_function_point`	Controls whether a step is forced at every point in the function.

`IterationAdaptiveDT` grows or shrinks the timestep based on the number of iterations taken to obtain a converged solution in the last converged step. The required `optimal_iterations` parameter controls the number of nonlinear iterations per timestep that provides optimal solution efficiency. If more iterations than

that are required to obtain a converged solution, the timestep may be too large, resulting in undue solution difficulty, while if fewer iterations are required, it may be possible to take larger timesteps to obtain a solution more quickly.

A second parameter, `iteration_window`, is used to control the size of the region in which the timestep is held constant. As shown in Figure [fig:adaptive_dt_criteria], if the number of nonlinear iterations for convergence is lower than (`optimal_iterations−iteration_window`), the timestep is increased, while if more than (`optimal_iterations+iteration_window`), iterations are required, the timestep is decreased. The `iteration_window` parameter is optional. If it is not specified, it defaults to $1/5$ the value specified for `optimal_iterations`.

The decision on whether to grow or shrink the timestep is based both on the number of nonlinear iterations and the number of linear iterations. The parameters mentioned above are used to control the optimal iterations and window for nonlinear iterations. The same criterion is applied to the linear iterations. Another parameter, `linear_iteration_ratio`, which defaults to 25, is used to control the optimal iterations and window for the linear iterations. These are calculated by multiplying `linear_iteration_ratio` by `optimal_iterations` and `iteration_window`, respectively.

To grow the timestep, the growth criterion must be met for both the linear iterations and nonlinear iterations. If the timestep shrinkage criterion is reached for either the linear or nonlinear iterations, the timestep is decreased. To control the timestep size only based on the number of nonlinear iterations, set `linear_iteration_ratio` to a large number.

If the timestep is to be increased or decreased, the amount of the increase or decrease in the timestep is controlled by the `growth_factor` and `cutback_factor` parameters, respectively. If a solution fails to converge when adaptive time stepping is active, a new attempt is made using a smaller timestep in the same manner as with the fixed timestep methods. The maximum and minimum timesteps can be optionally specified in the `Executioner` block using the `dtmax` and `dtmin` parameters, respectively.

In addition to controlling the timestep based on the iteration count, `IterationAdaptiveDT` also has an option to limit the timestep based on the behavior of a time-dependent function, optionally specified by providing the function name in `timestep_limiting_function`. This is typically a function that is used to drive boundary conditions of the model. The step is cut back if the change in the function from the previous step exceeds the value specified in `max_function_change`. This allows the step size to be changed to limit the change in the boundary conditions applied to the model over a step. In addition to that limit, the Boolean parameter `force_step_every_function_point` can be set to `true` to force a timestep at every point in a `PiecewiseLinear` function.
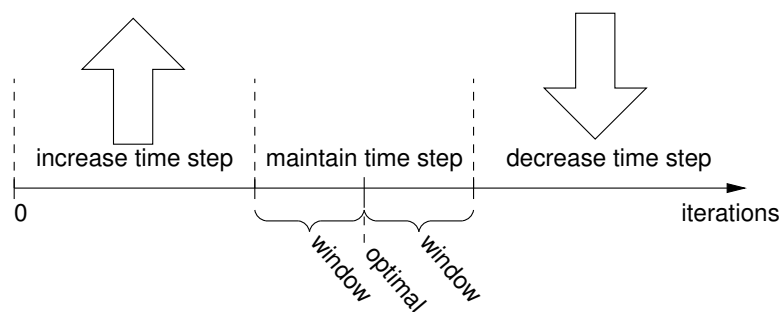
Figure 2: Criteria used to determine adaptive timestep size

4.6 PETSc Options

The PETSc solver used by MOOSE is very flexible and permits the use of multiple solution strategies. The full set of options provided by PETSc cannot be covered in detail here. The solver options used by any code that uses PETSc can be controlled with command line parameters. MOOSE-based codes additionally permit the specification of these PETSc options through two parameters: `petsc_options_iname` and `petsc_options_value` in the `Executioner` block. Using these options permits specifying the solver options in the input file so that they do not need to be specified at the command line, which can greatly simplify the process of running the code.

   The following PETSc options invoke the use of an algebraic multigrid preconditioner, which scales well in parallel and uses minimal memory.

```
[Executioner]
  ...
  petsc_options_iname = '-ksp_gmres_restart -pc_type -pc_hypre_type
                         -pc_hypre_boomeramg_max_iter'
  petsc_options_value = '201 hypre boomeramg 4'
  ...
[../]
```

   These options are recommended for most problems.

4.7 Quadrature

The order of the quadrature rule used for finite element integration can optionally be specified using a `[./Quadrature]` subblock within the `[Executioner]` block as follows:

```
[./Quadrature]
  type = <string>
  element_order = <string>
  order = <string>
  side_order = <string>
[../]
```

`type`	The type of quadrature used. Default is Gauss.
`element_order`	Order of quadrature on the elements.
`order`	Order of quadrature used.
`side_order`	Order of quadrature used on the sides.

4.8 Variables

The `Variables` block is where all of the primary solution variables are identified. The name of each variable is assigned according to the name of its subblock. Primary solution variables often include temperature (usually named `temp`) and displacement (usually named `disp_x`, `disp_y`, and `disp_z`).

```
[Variables]
  [./var1]
    order = <string>
```

```
    family = <string>
  [../]
  [./var2]
    order = <string>
    family = <string>
    initial_condition = <real>
    scaling = <real> (1)
  [../]
[]
```

`order`	The order of the variable. Typical values are `FIRST` and `SECOND`.
`family`	The finite element shape function family. A typical value is `LAGRANGE`.
`initial_condition`	Optional initial value to be assigned to the variable. Zero is assigned if this line is not present.
`scaling`	Amount to scale the variable during the solution process. This scaling affects only the residual and preconditioning steps and not the final solution values. This is sometimes helpful when solving coupled systems where one variable's residual is orders of magnitude different that the other variables' residuals.

4.9 Kernels

`Kernels` are used to evaluate integrals associated with a given term in a PDE. They often compute quantities based on functions, solution variables, auxiliary variables, and material properties. All `Kernels` act on blocks. If no block is specified, the `Kernel` will act on the entire model.

```
[Kernels]
  [./name]
    type = <kernel type>
    block = <string list>
    ...
  [../]
[]
```

`type`	Type of kernel.
`block`	List of blocks. Either block numbers or names.

4.9.1 TimeDerivative

Kernel for applying a time rate of change term ($\partial u / \partial t$) to the model.

```
  [./timederivative]
    type = TimeDerivative
```

```
    variable = <variable>
  [../]
```

type	TimeDerivative
variable	**Required**. Variable associated with this volume integral.

4.9.2 BodyForce

Kernel for applying an arbitrary body force to the model.

```
  [./bodyforce]
    type = BodyForce
    variable = <variable>
    value = <real> (0)
    function = <string> (1)
  [../]
```

type	BodyForce
variable	**Required**. Variable associated with this volume integral.
value	Constant included in volume integral. Multiplied by the value of `function` if present.
function	Function to be multiplied by `value` and used in the volume integral.

4.10 AuxVariables

The `AuxVariables` block is where the auxiliary variables are identified. Auxiliary variables are field variables that are not solved for in the equation system, but are used for storing the values of auxiliary quantities that are used in the simulation or for output. The name of each auxiliary variable is assigned according to the name of its subblock.

```
[AuxVariables]
  [./var1]
    order = <string>
    family = <string>
  [../]
  [./var2]
    order = <string>
    family = <string>
    initial_condition = <real>
  [../]
[]
```

order	The order of the variable. Typical values are CONSTANT, FIRST, and SECOND.

family	The finite element shape function family. Typical values are `MONOMIAL` and `LAGRANGE`.
initial_condition	Optional initial value to be assigned to the variable. Zero is assigned if this line is not present.

4.11 Materials

The `Materials` block is for specifying material properties and models.

```
[Materials]
  [./name]
    type = <material type>
    block = <string list>
    ...
  [../]
[]
```

type	Type of material model
block	List of blocks. Either block numbers or names.

4.12 Postprocessors

MOOSE `Postprocessors` compute a single scalar value at each timestep. These can be minimums, maximums, averages, volumes, or any other scalar quantity.

```
[Postprocessors]
  [./name]
    type = <postprocessor type>
    block = <string list>
    boundary = <string list>
    execute_on = <string list>
    outputs = <string>
    ...
  [../]
[]
```

type	Type of postprocessor
block	List of blocks. Either block numbers or names.
boundary	List of boundaries (side sets). Either boundary numbers or names.
execute_on	Set to (`nonlinear` | `linear` | `timestep_end` | `timestep_begin`) to execute at that moment.
outputs	Vector of output names where you would like to restrict the output of variable(s) associated with the postprocessor.

Most `Postprocessors` act on either boundaries or blocks. If no block or boundary is specified, the `Postprocessor` will act on the entire model. There are a few `Postprocessors` that act on specific nodes or elements within the finite element mesh.

4.12.1 ElementalVariableValue

In some cases it may be of interest to output an elemental variable value (e.g., stress) at a particular location in the model. This is accomplished by using the `ElementalVariableValue` postprocessor.

```
[./elementalvariablevalue]
  type = ElementalVariableValue
  elementid = <string>
  variable = <string>
[../]
```

type	ElementalVariableValue
elementid	**Required**. The global element id from the mesh to which this postprocessor is to be applied.
variable	**Required**. The variable whose value is output to this postprocessor for the given element.

4.12.2 NodalVariableValue

In order to obtain the value of a nodal variable at a particular location (i.e., temperature and displacement) a `NodalVariableValue` postprocessor is used. For example, this postprocessor is useful for obtaining the centerline temperature at the location of a thermocouple to compare against experimental data.

```
[./nodalvariablevalue]
  type = NodalVariableValue
  elementid = <string>
  scale_factor = <real>
  variable = <string>
[../]
```

type	NodalVariableValue
nodeid	**Required**. The global node id from the mesh to which this postprocessor is to be applied.
scale_factor	A scalar value to be multiplied by the value of the variable.
variable	**Required**. The variable whose value is output to this postprocessor for the given node.

4.12.3 NumNonlinearIterations

`NumNonlinearIterations` reports the number of nonlinear iterations in the just-completed solve.

```
[./numnonlineariters]
  type = NumNonlinearIterations
```

19

```
[../]
```

type	NumNonlinearIterations

4.12.4 PlotFunction

PlotFunction gives the value of the supplied function at the current time, optionally scaled with `scale_factor`.

```
[./plotfunction]
  type = PlotFunction
  function = <string>
  scale_factor = <real> (1)
[../]
```

type	PlotFunction
function	**Required**. The function to evaluate.
scale_factor	Scale factor to be applied to the function value.

4.12.5 SideAverageValue

SideAverageValue computes the area- or volume-weighted average of the named variable. It may be used, for example, to calculate the average temperature over a side set.

```
[./sideaveragevalue}
  type = SideAverageValue
  variable = <string>
[../]
```

type	SideAverageValue
variable	**Required**. The variable this Postprocessor acts on.

4.12.6 SideFluxIntegral

SideFluxIntegral computes the integral of the flux over the given boundary.

```
[./sidefluxintegral]
  type = SideFluxIntegral
  variable = <string>
  diffusivity = <string>
[../]
```

type	SideFluxIntegral
variable	**Required**. Variable to be used in the flux calculation.
diffusivity	**Required**. The diffusivity material property to be used in the calculation.

4.12.7 TimestepSize

`TimestepSize` reports the timestep size.

```
  [./dt]
    type = TimestepSize
  [../]
```

type	TimestepSize

4.13 VectorPostprocessors

`VectorPostprocessors` computes vectors of values and outputs them in comma separated files, one file for each VectorPostprocessor at each timestep. The parameter `csv` must be set to `true` in the `Outputs` block to allow the files to be output.

```
[VectorPostprocessors]
  [./name]
    type = <vectorpostprocessor type>
    execute_on = <string list>
    ...
  [../]
[]
```

type	Type of postprocessor
execute_on	Set to (`nonlinear` | `linear` | `timestep_end` | `timestep_begin`) to execute at that moment.

4.13.1 LineValueSampler

The `LineValueSampler` VectorPostprocessor samples the values of a variable along a user-specified line.

```
[./linevaluesampler]
  type = LineValueSampler
  variable = <string>
  start_point = <vector>
  end_point = <vector>
  num_points = <integer>
  sort_by = <x or y or z or id>
[../]
```

type	LineValueSampler
variable	**Required** The names of the variables that this VectorPostprocessor operates on.
start_point	**Required** The beginning of the line.
end_point	**Required** The ending of the line.

`num_points`	**Required** The number of points to sample along the line.
`sort_by`	**Required** What to sort the samples by.

4.14 Functions

4.14.1 Composite

The `Composite` function takes an arbitrary set of functions, provided in the `functions` parameter, evaluates each of them at the appropriate time and position, and multiplies them together. The function can optionally be multiplied by a scale factor, which specified using the `scale_factor` parameter.

```
[./composite]
  type = CompositeFunction
  functions = <string list>
  scale_factor = <real> (1.0)
[../]
```

`type`	CompositeFunction
`functions`	List of functions to be multiplied together.
`scale_factor`	Scale factor to be applied to resulting function. Default is 1.

4.14.2 ParsedFunction

The `ParsedFunction` function takes a mathematical expression in `value`. The expression can be a function of time (t) or coordinate (x, y, or z). The expression can include common mathematical functions. Examples include '4e4+1e2*t', 'sqrt(x*x+y*y+z*z)', and 'if(t<=1.0, 0.1*t, (1.0+0.1)*cos(pi/2*(t-1.0)) - 1.0)'. Constant variables may be used in the expression if they have been declared with `vars` and defined with `vals`. Further information can be found at `http://warp.povusers.org/FunctionParser/`.

```
[./parsedfunction]
  type = ParsedFunction
  value = <string>
  vals = <real list>
  vars = <string list>
[../]
```

`type`	ParsedFunction
`value`	**Required**. String describing the function.
`vals`	Values to be associated with variables in `vars`.
`vars`	Variable names to be associated with values in `vals`.

4.14.3 PiecewiseBilinear

The `PiecewiseBilinear` function reads a csv file and interpolates values based on the data in the file. The interpolation is based on x-y pairs. If `axis` is given, time is used as the y index. Either `xaxis` or `yaxis` or both may be given. Time is used as the other index if one of them is not given. If `radius` is given, `xaxis`

and `yaxis` are used to orient a cylindrical coordinate system, and the x-y pair used in the query will be the radial coordinate and time.

```
[./piecewiselinear]
  type = PiecewiseBilinear
  data_file = <string>
  axis = <0, 1, or 2 for x, y, or z>
  xaxis = <0, 1, or 2 for x, y, or z>
  yaxis = <0, 1, or 2 for x, y, or z>
  scale_factor = <real> (1.0)
  radial = <bool> (false)
[../]
```

type	PiecewiseBilinear
data_file	File holding the csv data.
axis	Coordinate direction to use in the function evaluation.
xaxis	Coordinate direction used for x-axis data.
yaxis	Coordinate direction used for y-axis data.
scale_factor	Scale factor to be applied to resulting function. Default is 1.
radial	Set to `true` if interpolation should be done along a radius rather than along a specific axis. Requires `xaxis` and `yaxis`.

4.14.4 PiecewiseConstant

The `PiecewiseConstant` function defines the data using a set of x-y data pairs. Instead of linearly interpolating between the values, however, the `PiecewiseConstant` function is constant when the abscissa is between the values provided by the user. The `direction` parameter controls whether the function takes the value of the abscissa of the user-provided point to the right or left of the value at which the function is evaluated.

```
[./piecewiseconstant]
  type = PiecewiseConstant
  x = <real list>
  y = <real list>
  xy_data = <real list>
  data_file = <string>
  format = <string> (rows)
  scale_factor = <real> (1.0)
  axis = <0, 1, or 2 for x, y, or z>
  direction = <string> (left)
[../]
```

type	PiecewiseConstant
x	List of x values for x-y data.
y	List of y values for x-y data.
xy_data	List of pairs of x-y data points.

`data_file`	Name of an file containing x-y data.
`format`	Format of x-y data in external file.
`scale_factor`	Scale factor to be applied to resulting function. Default is 1.
`axis`	Coordinate direction to use in the function evaluation. If not present, time is used as the function input.

4.14.5 PiecewiseLinear

The `PiecewiseLinear` function performs linear interpolations between user-provided pairs of x-y data. The x-y data can be provided in three ways. The first way is through a combination of the x and y parameters, which are lists of the x and y coordinates of the data points that make up the function. The second way is in the `xy_data` parameter, which is a list of pairs of x-y data that make up the points of the function. This allows for the function data to be specified in columns by inserting line breaks after each x-y data point. Finally, the x-y data can be provided in an external file containing comma-separated values. The file name is provided in `data_file`, and the data can be provided in either rows (default) or columns, as specified in the `format` parameter.

By default, the x-data corresponds to time, but this can be changed to correspond to x, y, or z coordinate with the `axis` line. If the function is queried outside of its range of x data, it returns the y value associated with the closest x data point.

```
[./piecewiselinear]
  type = PiecewiseLinear
  x = <real list>
  y = <real list>
  xy_data = <real list>
  data_file = <string> (rows)
  format = <string>
  scale_factor = <real> (1.0)
  axis = <0, 1, or 2 for x, y, or z>
[../]
```

`type`	PiecewiseLinear
`x`	List of x values for x-y data.
`y`	List of y values for x-y data.
`xy_data`	List of pairs of x-y data points.
`data_file`	Name of an file containing x-y data.
`format`	Format of x-y data in external file.
`scale_factor`	Scale factor to be applied to resulting function. Default is 1.
`axis`	Coordinate direction to use in the function evaluation. If not present, time is used as the function input.

4.15 BCs

The `BCs` block is for specifying various types of boundary conditions. Boundary conditions that are applicable to any physics are listed here.

```
[BCs]
```

```
  [./name]
    type = <BC type>
    boundary = <string list>
    ...
  [../]
[]
```

type	Type of boundary condition.
boundary	List of boundaries (side sets). Either boundary numbers or names.

4.15.1   DirichletBC

The `DirichletBC` boundary condition is used to directly specify the value of a solution variable. It can be applied to any solution variable.

```
  [./dirichletbc]
    type = DirichletBC
    variable = <variable>
    boundary = <string list>
    value = <real>
  [../]
```

type	DirichletBC
variable	**Required**. Primary variable associated with this boundary condition.
boundary	**Required**. List of boundary names or ids where this boundary condition will apply.
value	**Required**. Value to be assigned.

4.15.2   PresetBC

The `PresetBC` takes the same inputs as `DirichletBC` and also acts as a Dirichlet boundary condition. However, the implementation is slightly different. `PresetBC` causes the value of the boundary condition to be applied before the solve begins where `DirichletBC` enforces the boundary condition as the solve progresses. In certain situations, one is better than another.

4.15.3   FunctionDirichletBC

```
  [./functiondirichletbc]
    type = FunctionDirichletBC
    variable = <variable>
    boundary = <string list>
    function = <string>
  [../]
```

type	FunctionDirichletBC
variable	**Required**. Primary variable associated with this boundary condition.
boundary	**Required**. List of boundary names or ids where this boundary condition will apply.
function	**Required**. Function that will give the value to be applied by this boundary condition.

4.15.4 FunctionPresetBC

The `FunctionPresetBC` takes the same inputs as `FunctionDirichletBC` and also acts as a Dirichlet boundary condition. However, the implementation is slightly different. `FunctionPresetBC` causes the value of the boundary condition to be applied before the solve begins where `FunctionDirichletBC` enforces the boundary condition as the solve progresses. In certain situations, one is better than another.

4.16 AuxKernels

`AuxKernels` are used to compute values for `AuxVariables`. They often compute quantities based on functions, solution variables, and material properties. `AuxKernels` can apply to blocks or boundaries. If no block or boundary is specified, the `AuxKernel` applies to the entire model.

```
[AuxKernels]
  [./name]
    type = <AuxKernel type>
    block = <string list>
    boundary = <string list>
    ...
  [../]
[]
```

type	Type of auxiliary kernel.
block	List of blocks. Either block numbers or names.
boundary	List of boundaries (side sets). Either boundary numbers or names.

4.16.1 MaterialRealAux

The `MaterialRealAux` AuxKernel is used to output material properties. Typically, the AuxVariable computed by `MaterialRealAux` will be an element-level, constant variable. The computed value will be the volume-averaged quantity over the element.

```
  [./materialrealaux]
    type = MaterialRealAux
    property = <material property>
    variable = <variable>
  [../]
```

type	`MaterialRealAux`
property	**Required**. Name of material property.
variable	**Required**. Name of `AuxVariable` that will hold result.

4.17 Constraints

The `Constraints` block is for specifying various types of constraints. Constraints that are applicable to any physics are listed here.

```
[Constraints]
  [./name]
    type = <Constraint type>
    ...
  [../]
[]
```

type	Type of constraint.

4.17.1 EqualValueBoundaryConstraint

The `EqualValueBoundaryConstraint` Constraint is used to constrain a variable to have the same value on a boundary or set of nodes.

```
[./equalvalueboundaryconstraint]
  type = EqualValueBoundaryConstraint
  variable = <variable>
  master = <integer>
  slave = <string>
  slave_node_ids = <integer list>
  formulation = <Penalty or Kinematic> (Penalty)
  penalty = <real>
[../]
```

type	EqualValueBoundaryConstraint
variable	**Required**. Primary variable associated with this boundary condition.
master	The id of the master node. If no id is provided, first node of slave set is chosen.
slave	The boundary id associated with the slave side. Must use either this or `slave_node_ids`.
slave_node_ids	The ids of the slave nodes. Must use either this or `slave`.
formulation	Formulation used to calculate constraint. One of Penalty or Kinematic.
penalty	**Required** The penalty stiffness value to be used in the constraint.

4.18 UserObjects

UserObjects in MOOSE provide a capability to implement a wide variety of capabilities that do not readily fit into the interfaces provided by other types of MOOSE objects. These are defined within the UserObjects block in the input file.

```
[UserObjects]
  [./name]
    type = <UserObject type>
    ...
  [../]
[]
```

type	Type of user object.

4.19 Outputs

The Outputs block lists parameters that control the frequency and type of results files produced. It is possible to create multiple output objects each outputting at different intervals, or different variables, or varying file types. The Outputs system is very complex and enables a large amount of customization. This section will highlight the capabilities of the system.

4.19.1 Basic Input File Syntax

To enable output an input file must contain an Outputs block. The simplest method for enabling output is to utilize the shortcut syntax as shown below, which enables the Console output (prints to screen) and Exodus output for writing data to a file.

```
[Outputs]
  console = true   #output to the screen with default settings
  exodus = true    #output to ExodusII file with default settings
[]
```

4.19.2 Advanced Syntax

To take full advantage of the output system the use of subblocks is required. For example, the input file snippet below is **exactly** equivalent, including the subblock names, to the snippet shown above that utilizes the shortcut syntax.

```
[Outputs]
  [./console]
    type = Console   #output to the screen with default settings
  [../]
  [./exodus]
    type = Exodus    #output to ExodusII file with default settings
  [../]
[]
```

However, the subblock syntax allows for increased control over the output and allows for multiple outputs of the same type to be specified. For example, the following creates two Exodus outputs, one outputting the a mesh at every timestep including the initial condition the other outputs every 3 timesteps without the initial condition. Additionally, performance logging was enabled for `Console` output.

```
[Outputs]
  execute_on = 'timestep_end' # Limit the output to timestep end (no initial state)
  [./console]
    type = Console
    perf_log = true       # enable performance logging
  [../]
  [./exodus]
    type = Exodus
    execute_on = 'initial timestep_end' # output the initial state
  [../]
  [./exodus_3]                # create a second [Exodus II][1] output
                              # that utilizes a different output interval
    type = Exodus
    file_base = exodus_3  # set the file base
                              # (the extension is automatically applied)
    interval = 3              # only output every third step
  [../]
[]
```

4.19.3 Common Output Parameters

In addition to allowing for shortcut syntax, the `Outputs` block also supports common parameters. For example, the parameter `execute_on` may be specified outside of individual subblocks, indicating that all subblocks should output at the specified time. If within a subblock the parameter is given a different value, the subblock parameter takes precedence. The input file snippet below demonstrate the usage of a common values as well as the use of multiple output blocks.

```
[Outputs]
  execute_on = 'timestep_end' # disable the output of initial state
  vtk = true                   # output VTK file with default setting
  [./exodus]
    type = Exodus
    execute_on = 'initial' # this file will contain ONLY the initial state
  [../]
  [./exodus_displaced]
    type = Exodus
    file_base = displaced
    use_displaced = true
    interval = 3             # this file will only output every third time step
  [../]
[]
```

4.19.4 File Output Names

The default naming scheme for output files utilizes the input file name (e.g., input.i) with a suffix that differs depending on how the output is defined:

- output files created using the shortcut syntax have an _out suffix

- subblocks use the actual subblock name as the suffix.

For example, if the input file (input.i) contained the following [Outputs] block, two files would be created: `input_out.e` and `input_other.e`.

```
[Outputs]
  console = true
  exodus = true     # creates input_out.e
  [./other]         # creates input_other.e
    type = Exodus
    interval = 2
  [../]
[]
```

4.20 Dampers

Dampers are used to decrease the attempted change to the solution with each nonlinear step. This can be useful in preventing the solver from changing the solution dramatically from one step to the next. This may prevent, for example, the solver from attempting to evaluate negative temperatures.

The MaxIncrement damper is commonly used.

4.20.1 MaxIncrement

The MaxIncrement damper limits the change of a variable from one nonlinear step to the next.

```
[Dampers]
  [./maxincrement]
    type = MaxIncrement
    max_increment = <real>
    variable = <string>
  [../]
[]
```

type	MaxIncrement
max_increment	**Required**. The maximum change in solution variable allowed from one nonlinear step to the next.
variable	**Required**. Variable that will not be allowed to change beyond max_increment from nonlinear step to nonlinear step.

5   Thermal Governing Equations and Associated Commands

The energy balance is given in terms of the heat conduction equation

$$\rho C_p \frac{\partial T}{\partial t} + \nabla \cdot \boldsymbol{q} = 0,$$

where T, ρ and C_p are the temperature, density and specific heat, respectively. The heat flux is given as

$$\boldsymbol{q} = -k\nabla T,$$

where k denotes the thermal conductivity of the material.

5.1   Kernels

5.1.1   Heat Conduction

Kernel for diffusion of heat or divergence of heat flux.

```
[./heatconduction]
  type = HeatConduction
  variable = <variable>
[../]
```

type	HeatConduction
variable	**Required**. Variable name corresponding to the heat conduction equation. Typically `temp`.

5.1.2   Heat Conduction Time Derivative

Kernel for $\rho C_p \partial T / \partial t$ term of the heat equation.

```
[./heatconductiontimederivative]
  type = HeatConductionTimeDerivative
  variable = <variable>
[../]
```

type	HeatConductionTimeDerivative
variable	**Required**. Variable name corresponding to the heat conduction equation. Typically `temp`.

5.1.3   Heat Source

The `HeatSource` kernel applies a volumetric heat source to specified blocks within the model. Built on the `BodyForce` kernel's code, the `HeatSource` kernel provides a more relevant name for easier input-file specification.

```
[./heatsource]
  type = HeatSource
```

```
  variable = <variable>
  value = <real> (1)
  function = <string> (1)
  block = <string list>
[../]
```

`type`	HeatSource
`variable`	**Required**. The variable associated with the heat source.
`value`	Value of the heat source; will be multiplied by the optional `function`.
`function`	The function describing the volumetric heat source.
`block`	The list of block id's (SubdomainID) to which the heat source will be applied.

5.2 Materials

5.2.1 HeatConductionMaterial

`HeatConductionMaterial` is a general-purpose material model for heat conduction. It sets the thermal conductivity and specific heat at integration points.

```
[heatconductionmaterial]
  type = HeatConductionMaterial
  temp = <string>
  thermal_conductivity = <real>
  thermal_conductivity_temperature_function = <string>
  specific_heat = <real>
  specific_heat_temperature_function = <string>
  block = <string list>
[../]
```

`type`	HeatConductionMaterial
`temp`	Name of temperature variable. Typically `temp`.
`thermal_conductivity`	Coefficient of thermal conductivity.
`thermal_conductivity_temperature_function`	Function describing thermal conductivity as a function of temperature.
`specific_heat`	Specific heat capacity.
`specific_heat_temperature_function`	Function describing specific heat capacity as a function of temperature.
`block`	List of blocks this material applies to.

5.3 BCs

5.3.1 ConvectiveFluxFunction

The `ConvectiveFluxFunction` boundary condition determines the value on a boundary based upon the heat transfer coefficient of the fluid on the outside of boundary and far-field temperature.

```
[./convectivefluxFunction]
  type = ConvectiveFluxFunction
  variable = <variable>
  boundary = <string list>
  T_infinity= <string>
  coefficient = <real>
  coefficient_function = <string>
[../]
```

type	ConvectiveFluxFunction
variable	**Required**. Primary variable associated with this boundary condition.
boundary	**Required**. List of boundary names or ids where this boundary condition will apply.
T_infinity	**Required**. The name of the function describing the far-field temperature.
coefficient	**Required**. The heat transfer coefficient of the fluid in contact with the boundary. If `coefficient_function` is provided this coefficient multiplies the function.
coefficient_function	Function defining the heat transfer coefficient as a function of time.

6 Mechanical Governing Equation, Kinematics, and Associated Commands

Momentum conservation is prescribed assuming static equilibrium at each time increment using Cauchy's equation,

$$\nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{f} = 0,$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor and $\boldsymbol{f}$ is the body force per unit mass (e.g. gravity). The displacement field u, which is the primary solution variable, is connected to the stress field via the strain, through a constitutive relation.

For geometrically linear analysis, the strain ϵ is defined as $1/2[\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T]$. Furthermore, with a linear elastic constitutive model, the stress is simply $C \, \epsilon$. We now outline our approach for nonlinear analysis. We follow the approach in [1] and the software package FMA3D [2].

We begin with a complete set of data for step n and seek the displacements and stresses at step $n+1$. We first compute an incremental deformation gradient,

$$\hat{\boldsymbol{F}} = \frac{\partial x^{n+1}}{\partial x^n}.$$

With $\hat{\boldsymbol{F}}$, we next compute a strain increment that represents the rotation-free deformation from the configuration at n to the configuration at $n+1$. Following [1], we seek the stretching rate $\boldsymbol{D}$:

$$\begin{aligned}
\boldsymbol{D} &= \frac{1}{\Delta t} \log(\hat{\boldsymbol{U}}) \\
&= \frac{1}{\Delta t} \log\left(\text{sqrt} \left(\hat{\boldsymbol{F}}^T \hat{\boldsymbol{F}} \right) \right) \\
&= \frac{1}{\Delta t} \log\left(\text{sqrt} \left(\hat{\boldsymbol{C}} \right) \right).
\end{aligned}$$

Here, $\hat{\boldsymbol{U}}$ is the incremental stretch tensor, and $\hat{\boldsymbol{C}}$ is the incremental Green deformation tensor. Through a Taylor series expansion, this can be determined in a straightforward, efficient manner. $\boldsymbol{D}$ is passed to the constitutive model as an input for computing $\boldsymbol{\sigma}$ at $n+1$.

The next step is computing the incremental rotation, $\hat{\boldsymbol{R}}$ where $\hat{\boldsymbol{F}} = \hat{\boldsymbol{R}} \hat{\boldsymbol{U}}$. Like for $\boldsymbol{D}$, an efficient algorithm exists for computing $\hat{\boldsymbol{R}}$. It is also possible to compute these quantities using an eigenvalue/eigenvector routine.

With $\boldsymbol{\sigma}$ and $\hat{\boldsymbol{R}}$, we rotate the stress to the current configuration.

6.1 Temperature-dependent thermal expansion

Both the base metal and the liner in RPVs have temperature-dependent elastic properties and thermal expansion coefficients. The thermal expansion coefficient can be specified as an instantaneous thermal expansion coefficient, which defines the thermal strain increment under an infinitesimal change in the temperature. Using the notation of [3], for an isotropic material, this instantaneous thermal expansion coefficient at a given temperature, T is referred to as $\alpha_{(T)}$, and the instantaneous thermal strain, ϵ^{th} is defined as:

$$d\epsilon^{th} = \alpha_{(T)} dT.$$

Alternatively, the thermal expansion can be expressed as a mean coefficient of thermal expansion, $\bar{\alpha}_{(T)}$ at a given temperature relative to the strain at a reference temperature, T_{ref}:

$$\bar{\alpha}_{(T)} = \frac{L_{(T)} - L_{(T_{ref})}}{L_{(T_{ref})}(T - T_{ref})}$$

34

Because mean thermal expansion can be readily measured experimentally, it is common to express thermal expansion for materials in this manner, as is the case for RPV materials.
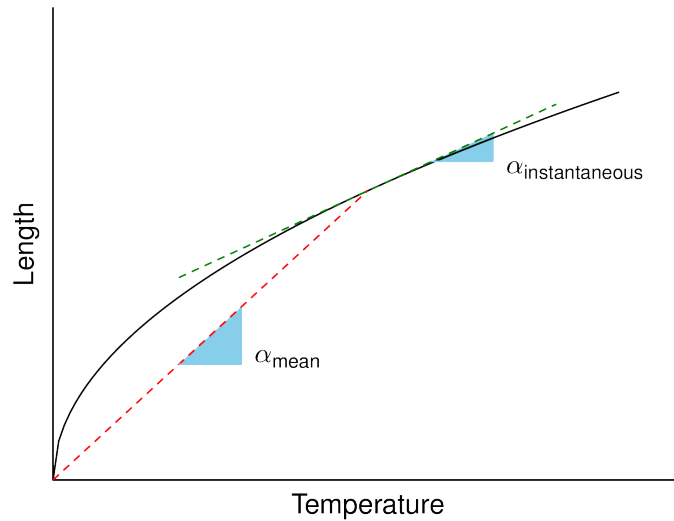
Figure 3: Instantaneous and mean coefficients of thermal expansion

To permit the use of data in its most available form and facilitate comparisons with other codes, Grizzly allows temperature-dependent thermal expansion data to be provided in either format. If the data is provided in the mean form, a reference temperature must be specified, and it does not need to coincide with the stress-free temperature. Indeed, it is often assumed that the thermal strain in the nuclear reactor components is zero at the operating temperature, and this can be achieved in Grizzly by assigning the operating temperature as the stress-free temperature. The method outlined in [3] is employed to calculate the incremental thermal strain when the reference and stress-free temperatures differ. Regression tests have been developed to demonstrate that these two methods produce the same results for equivalent mean and instantaneous thermal expansion functions, including usage with fracture integrals.

6.2   Kernels

6.2.1   SolidMechanics

The `SolidMechanics` block specifies inputs for the divergence of stress as part of the equations of solid mechanics. The divergence of stress is a `Kernel` in MOOSE nomenclature. The `SolidMechanics` block informs MOOSE of the divergence kernels but is not placed inside the `Kernels` block in the input file.

```
[SolidMechanics]
  [./solidmechanics]
    disp_x = <variable>
    disp_y = <variable>
    disp_z = <variable>
    disp_r = <variable>
    temp = <variable>
  [../]
[]
```

`disp_x`	Variable name for displacement variable in x direction. Typically `disp_x`.
`disp_y`	Variable name for displacement variable in y direction. Typically `disp_y`.
`disp_z`	Variable name for displacement variable in z direction. Typically `disp_z` for 3D and `disp_y` for axisymmetric models.
`disp_r`	Variable name for displacement variable in radial direction for axisymmetric or spherically symmetric cases. Typically `disp_x`.
`temp`	Variable name for temperature variable. Necessary for thermal expansion. Typically `temp`.

6.2.2 Gravity

Gravity may be applied to the model with this kernel. The required density is computed and provided internally given inputs in the `Materials` block.

```
[./gravity]
  type = Gravity
  variable = <variable>
  value = <real> (0)
[../]
```

`type`	`Gravity`
`variable`	**Required**. Variable name corresponding to the displacement direction in which the gravity load should be applied.
`value`	Acceleration of gravity. Typically -9.81 (m/s^2).

6.3 AuxKernels

6.3.1 MaterialTensorAux

The `MaterialTensorAux` AuxKernel is used to output quantities related to second-order tensors used as material properties. Stress and strain are common examples of these tensors. The AuxKernel allows output of specific tensor entries or quantities computed from the entire tensor. Typically, the AuxVariable computed by `MaterialTensorAux` will be an element-level, constant variable. By default, the computed value is a volume-averaged quantity over the element.

```
[./materialtensoraux]
  type = MaterialTensorAux
  tensor = <material property tensor>
  variable = <variable>
  index = <integer>
  quantity = <string>
  point1 = <vector> (0, 0, 0)
```

```
  point2 = <vector> (0, 1, 0)
  direction = <vector>
  qp_select = <integer> (0, 1, ..., n)
[../]
```

type	MaterialTensorAux
tensor	**Required**. Name of second-order tensor material property. A typical second-order tensor material property is `stress`.
variable	**Required**. Name of `AuxVariable` that will hold result.
index	Index into tensor, from 0 to 5 (xx, yy, zz, xy, yz, zx). Either `index` or `quantity` must be specified.
quantity	One of `VonMises`, `PlasticStrainMag`, `Hydrostatic`, `Direction`, `Hoop`, `Radial`, `Axial`, `MaxPrincipal`, `MedPrincipal`, `MinPrincipal`, `FirstInvariant`, `SecondInvariant`, `ThirdInvariant`, `TriAxiality`, or `VolumetricStrain`. Either `index` or `quantity` must be specified.
point1	Start point for axis used to calculate some material tensor quantities.
point2	End point for axis used to calculate some material tensor quantities.
direction	Direction vector used to calculate some material tensor quantities.
qp_select	Output the value at the specified integration point number rather than volume averaging.

6.4 Materials

6.4.1 Elastic

The `Elastic` model is a simple hypo-elastic model.

```
[./elastic]
  type = Elastic
  disp_x = <string>
  disp_y = <string>
  disp_z = <string>
  disp_r = <string>
  temp = <string>
  youngs_modulus = <real>
  youngs_modulus_function = <string>
  poissons_ratio = <real>
  poissons_ratio_function = <string>
  thermal_expansion = <real> (0)
  thermal_expansion_function = <string>
  stress_free_temperature = <real>
  thermal_expansion_function_type = <string> (Mean)
```

```
    thermal_expansion_function = <string>
    thermal_expansion_reference_temperature = <real>
    block = <string list>
  [../]
```

`type`	`Elastic`
`disp_x`	Variable name for displacement variable in x direction. Typically `disp_x`.
`disp_y`	Variable name for displacement variable in y direction. Typically `disp_y`.
`disp_z`	Variable name for displacement variable in z direction. Typically `disp_z` for 3D and `disp_y` for axisymmetric models.
`disp_r`	Variable name for displacement variable in radial direction for axisymmetric or spherically symmetric cases. Typically `disp_x`.
`temp`	Name of temperature variable. Typically `temp`.
`youngs_modulus`	Young's modulus.
`youngs_modulus_function`	Function describing Young's modulus as a function of temperature.
`poissons_ratio`	Poisson's ratio.
`poissons_ratio_function`	Function describing Poisson's ratio as a function of temperature.
`thermal_expansion`	Coefficient of thermal expansion.
`stress_free_temperature`	The stress-free temperature. If not specified, the initial temperature is used.
`thermal_expansion_function_type`	Type of thermal expansion data. Options are Mean or Instantaneous.
`thermal_expansion_function`	Function describing thermal expansion as a function of temperature.
`thermal_expansion_reference_temperature`	Reference temperature for mean thermal expansion function.
`block`	List of blocks this material applies to.

6.5 Density

The `Density` model creates a material property named 'density'. If coupled to displacement variables, the model adjusts density based on deformation.

```
  [./density]
    type = Density
    disp_x = <string>
    disp_y = <string>
    disp_z = <string>
    disp_r = <string>
    density = <real>
  [../]
```

type	Density
disp_x	Variable name for displacement variable in x direction. Typically disp_x.
disp_y	Variable name for displacement variable in y direction. Typically disp_y.
disp_z	Variable name for displacement variable in z direction. Typically disp_z for 3D and disp_y for axisymmetric models.
disp_r	Variable name for displacement variable in radial direction for axisymmetric or spherically symmetric cases. Typically disp_x.
density	**Required**. Density.

6.6   VectorPostprocessors

6.6.1   LineMaterialSymmTensorSampler

LineMaterialSymmTensorSampler samples stress tensor quantities along a user-specified line.

```
[./linematerialsymmtensorsampler]
  type = LineMaterialSymmTensorSampler
  start = <vector>
  end = <vector>
  property = <string>
  quantity = <string>
  index = <integer>
  point1 = <vector> (0, 0, 0)
  point2 = <vector> (0, 1, 0)
  direction = <vector>
  sort_by = <x or y or z or id>
  block = <string list>
[../]
```

type	LineMaterialSymmTensorSampler
start	**Required** The beginning of the line.
end	**Required** The ending of the line.
property	**Required** Name of second-order tensor material property. A typical second-order tensor material property is stress.
index	Index into tensor, from 0 to 5 (xx, yy, zz, xy, yz, zx). Either index or quantity must be specified.
quantity	One of VonMises, PlasticStrainMag, Hydrostatic, Direction, Hoop, Radial, Axial, MaxPrincipal, MedPrincipal, MinPrincipal, FirstInvariant, SecondInvariant, ThirdInvariant, TriAxiality, or VolumetricStrain. Either index or quantity must be specified.

`direction`	Direction vector used to calculate some material tensor quantities.
`point1`	Start point for axis used to calculate some material tensor quantities.
`point2`	End point for axis used to calculate some material tensor quantities.
`sort_by`	**Required** What to sort the samples by.
`block`	List of blocks this vectorpostprocessor applies to.

6.7 BCs

6.7.1 Pressure

The `Pressure` boundary condition uses two levels of nesting within the `BCs` block. This allows the pressure to be applied properly in all coordinate directions although it is specified one time only.

```
[./Pressure]
  [./pressure]
    boundary = <string list>
    factor = <real> (1)
    function = <string>
  [../]
[../]
```

`boundary`	**Required**. List of boundary names or ids where this boundary condition will apply.
`factor`	Magnitude of pressure to be applied. If `function` is also given, `factor` is multiplied by the output of the function and then applied as the pressure.
`function`	Function that will give the value to be applied by this boundary condition.

7 Submodeling

Grizzly includes capabilities for solution transfer that facilitate submodeling. Typically the global model is the whole or part of a vessel and the submodel is a smaller region containing a flaw. Solutions stored in an ExodusII or XDA file can be used as input to a submodel using a `SolutionUserObject`. The mesh of the submodel must fit inside the physical domain of the global model. Advanced options of `SolutionUserObject` allow users to scale, translate and rotate the submodel. A solution function is required to interpolate in time and space the data from `SolutionUserObject`. The solution function to use depends on the type of solution mapping needed.

7.1 UserObjects

7.1.1 SolutionUserObject

A solution user object reads a variable from a mesh in one simulation to another. To use a `SolutionUserObject` three additional parameters are required, an `AuxVariable`, a `Function` and an `AuxKernel`. The `AuxVariable` represents the variable to be read by the solution user object. The `SolutionUserObject` is set up to read the old output file. A `SolutionFunction` is required to interpolate in time and space the data from the `SolutionUserObject`. Finally, the `FunctionAux` is required that will query the function and write the value into the `AuxVariable`. An example of what additions are required to the input file is shown below:

```
[AuxVariables]
  [./temp]
  [../]
[]

[Functions]
  [./interpolated_temp]
    type = SolutionFunction
    from_variable = 'temp'
    solution = read_thermo_solution
  [../]
[]

[UserObjects]
  [./read_thermo_solution]
    type = SolutionUserObject
    mesh = 'temp_from_another_simulation.e'
    execute_on = 'residual'
    nodal_variables = 'temp'
  [../]
[]

[AuxKernels]
  [./interp_temp]
    type = FunctionAux
    variable = 'temp'
    function = 'interpolated_temp'
  [../]
```

[]

Note that in the `SolutionUserObject` subblock that the mesh parameter is **required**.

7.2 Functions

7.2.1 SolutionFunction

If the global model and submodel have the same dimensionality, the `SolutionFunction` can be used as a forcing function for a boundary condition. The `SolutionFunction` uses a `SolutionUserObject` to read the data from a results file from a previous run. An example of how this is achieved in the input file is shown below:

```
[UserObjects]
  [./soln]
    type = SolutionUserObject
    mesh = 'global_model_out.e'
    system_variables = 'disp_x disp_y'
    timestep = 1
  [../]
[]
[Functions]
  [./bc_func_x]
    type = SolutionFunction
    solution = soln
    from_variable = disp_x
  [../]
[]
[BCs]
  [./x_soln_bc]
    type = FunctionDirichletBC
    variable = disp_x
    boundary = '1 2 3 4 5 6'
    function = bc_func_x
  [../]
[]
```

mesh	**Required.** The name of the mesh file containing the global model solution. Must be ExodusII or XDA file.
system_variables	The name of the nodal and elemental variables from the file you want to use for values.
timestep	Index of the single timestep used or "LATEST" for the last timestep (ExodusII only). If not supplied, time interpolation will occur.
solution	**Required.** The `SolutionUserObject` to extract data from.
from_variable	The name of the variable in the file that is to be extracted.

7.3 Axisymmetric2D3DSolutionFunction

If a two-dimensional axisymmetric model is used as the global model, the solution can be mapped onto a two- or three-dimensional submodel using `Axisymmetric2D3DSolutionFunction`. This function then replaces `SolutionFunction` in the example above.

```
[Functions]
  [./bc_func_x]
    type = Axisymmetric2D3DSolutionFunction
    solution = soln
    3d_axis_point1 = '0.0 0.0 0.0'
    3d_axis_point2 = '0.0 0.0 1.0'
    from_variables = 'disp_x disp_y'
    component = 0
    axial_dimension_ratio = 200.0
  [../]
[]
```

solution	**Required.** The `SolutionUserObject` to extract data from.
2d_axis_point1	Start point for axis of symmetry for the 2d model.
2d_axis_point2	End point for axis of symmetry for the 2d model.
3d_axis_point1	Start point for axis of symmetry for the 3d model.
3d_axis_point2	End point for axis of symmetry for the 3d model.
from_variables	The name of the variables in the file that are to be extracted, in x, y order if they are vector components.
component	Component of the variable to be computed if it is a vector.
axial_dimension_ratio	Ratio of the axial dimension in the 3d model to that in the 2d model. Optionally permits the 3d model to be larger than the 2d model in that dimension, and scales vector solutions in that direction by this factor.

8 Theory and Commands for Reactor Pressure Vessel Analysis

8.1 Fluence Map

Grizzly includes a capability to apply a neutron fluence map to the interior surface of the vessel. Inside the vessel wall, the fluence is calculated using the equation accepted by the NRC for fast neutron attenuation in RPV steel [4]. The fluence f at a given distance x (in inches) from the inner surface is expressed as a function of that distance and the fluence at the inner surface, f_i:

$$f(x) = f_i \exp(-0.24 * x).$$

The calculated fluence can be used in the EONY model presented below to calculate the transition-temperature shift for every location in the vessel, which in turn can provide the level of embrittlement in the fracture toughness given by the master curve model.
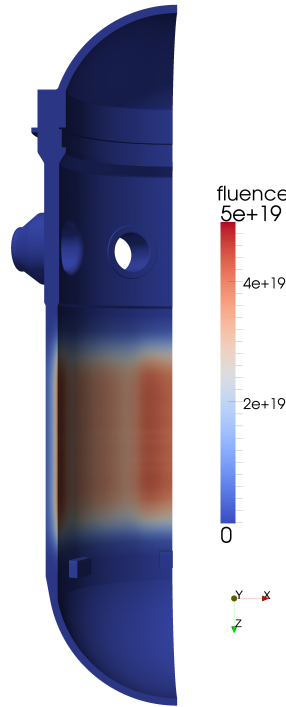

Figure 4: Attenuation through the thickness of the wall computed using `FluenceFunction` and a map of the fluence on the inner surface of the RPV.

The fluence is computed in the function `FluenceFunction` and can be output as an `AuxVariable`. `FluenceFunction` requires two data files describing the axial and azimuthal variation, respectively, of the fluence at the interior surface of the vessel. The second column in the data files gives the variation in fluence. In the file describing the axial variation of fluence, the first column is the height (in inches). The parameter `axis1` determines where the axial fluence profile will start and the vector from `axis1` to `axis2` defines the axial direction. In the file describing the azimuthal variation, the first column is the angle within a 90 degree sector of the RPV. The angle is defined relative to the position determined by the parameter `azimuth0`, which gives the position of the azimuthal origin in relation to the axial axis defined by the points `axis1` and `axis2`. The parameter `inner_radius` gives the radial distance from the center of the RPV at which the fluence map should be applied. At radial distances larger than `inner_radius`, the attenuated fluence is computed

using the formula presented above. In the end, the fluence is a product of the axial and azimuthal variation, respectively, the attenuation, and the `scale_factor` provided in the input block:

$$\phi = (\text{axial value}) \times (\text{azimuthal value}) \times (\text{scale factor}) \times (\text{attenuation}).$$

```
[AuxVariables]
  [./fluence]
    order = FIRST
    family = LAGRANGE
  [../]
[]

[Functions]
  [./flu_fun]
    type = FluenceFunction
    axial_file = <string>
    azimuthal_file = <string>
    axis1 = <vector>
    axis2 = <vector>
    azimuth0 = <vector>
    inner_radius = <real>
    scale_factor = <real>
  [../]
[]

[AuxKernels]
  [./fluence]
    type = FunctionAux
    function = flu_fun
    variable = fluence
    execute_on = initial
  [../]
[]
```

`axial_file`	**Required.** File with the fluence vs axial position
`azimuthal_file`	**Required.** File with the fluence vs azimuthal position
`axis1`	**Required.** First point on axis of rotation
`axis2`	**Required.** Second point on axis of rotation
`azimuth0`	**Required.** Point at azimuthal origin
`inner_radius`	**Required.** Inner radius of vessel (in meters)
`scale_factor`	Scale factor to be applied to the fluence

8.2 EONY Embrittlement Model

The EONY model by Eason, Odette, Nanstad and Yamamoto [5] is an empirically calibrated model based on U.S. nuclear reactor surveillance data for the embrittlement of RPV steel. It has been accepted for regulation and is incorporated in the U.S. NRC Regulations on Pressurized Thermal Shock, 10CFR50.61a. The

model predicts the transition-temperature shift in a RPV steel Charpy V-notch test as a function of fast neutron fluence, while considering the effects of flux, irradiation temperature, and chemical composition of the material. In this model, TTS is calculated as a function of two terms due to the matrix features (MF) and copper-rich precipitates (CRP):

$$\text{TTS} = \text{MF term} + \text{CRP term}$$

The MF term is calculated as

$$\text{MF term} = A(1 - 0.001718T_i)(1 + 6.13PMn^{2.47})\sqrt{\phi t_e}$$

where

$$A = \begin{cases} 1.140 \times 10^{-7} & \text{for forgings} \\ 1.561 \times 10^{-7} & \text{for plates} \\ 1.417 \times 10^{-7} & \text{for welds,} \end{cases}$$

T_i is the irradiation temperature (F), P is the weight % phosphorous content, Mn is the weight % manganese content, and ϕt_e is the fast neutron fluence (n/cm^2). The CRP term is calculated as:

$$\text{CRP term} = B(1 + 3.77Ni^{1.191})f(Cu_e, P)g(Cu_e, Ni, \phi t_e)$$

where

$$B = \begin{cases} 102.3 & \text{for forgings} \\ 102.5 & \text{for plates in non-CE mfg. vessels} \\ 135.2 & \text{for plates in CE mfg. vessels} \\ 155.0 & \text{for welds} \\ 128.2 & \text{for SRM plates,} \end{cases}$$

$$Cu_e = \begin{cases} 0 & \text{for } Cu \leq 0.072 \text{ wt\%} \\ \min(Cu, \max(Cu_e)) & \text{for } Cu > 0.072 \text{ wt\%,} \end{cases}$$

Cu_e is the weight % copper content, Ni is the weight % nickel content,

$$\max(Cu_e) = \begin{cases} 0.243 & \text{for typical } (Ni \leq 0.5) \text{ Linde 80 welds} \\ 0.301 & \text{for all other materials,} \end{cases}$$

$$f(Cu_e, P) = \begin{cases} 0 & \text{for } Cu \leq 0.072 \\ (Cu_e - 0.072)^{0.668} & \text{for } Cu > 0.072 \text{ and } P \leq 0.008 \\ (Cu_e - 0.072 + 1.359(P - 0.008))^{0.668} & \text{for } Cu > 0.072 \text{ and } P > 0.008 \end{cases}$$

and

$$g(Cu_e, Ni, \phi t_e) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{\log_{10}(\phi t_e) + 1.139Cu_e - 0.448Ni - 18.120}{0.629}\right).$$

Because it is calibrated to experimental data, this model provides reasonable predictions of the effect of embrittlement within the range of the data used to calibrate it. As acknowledged by its authors, it is

only applicable for limited fluence levels. The authors are working to provide an authoritative limit for its applicability, but provide a preliminary suggestion that it be limited to fluences of 5×10^{19} n/cm^2. Ranges and mean values of all the independent variables in the calibration database are provided in table 2 of reference [5].

The TTS predicted by the EONY model is calculated using the `AuxKernel` `EONYAux`, which outputs it as a material property field in the model. Typically the fluence variable is calculated in `FluenceFunction` and is a function of location in the vessel.

```
[./EONYAux]
  type = EONYAux
  variable = <string>
  fluence = <string>
  block = <string list>
  A = <real>
  B = <real>
  Cu = <real>
  Mn = <real>
  Ni = <real>
  P = <real>
  irradiation_time = <real>
  irradiation_temp = <real>
[../]
```

`variable`	**Required.** Name of `AuxVariable` that will hold transition temperature shift result.
`fluence`	**Required.** Variable name for fluence variable.
`block`	**Required.** List of blocks.
`A`	**Required.** A parameter.
`B`	**Required.** B parameter.
`Cu`	**Required.** Weight percentage of Cu.
`Mn`	**Required.** Weight percentage of Mn.
`Ni`	**Required.** Weight percentage of Ni.
`P`	**Required.** Weight percentage of P.
`irradiation_time`	**Required.** Irradiation time.
`irradiation_temp`	**Required.** Irradiation temperature (in degrees Fahrenheit).

8.3 Fracture Toughness From Master Curve

The master curve approach is typically used to obtain the DBTT in steels [6], [7]. It is based on fitting the experimentally obtained scattered fracture toughness data with the modified Weibull distribution function

$$P_f = 1 - \exp\left[-\left(\frac{K_J - 20}{K_0 - 20} \right)^4 \right]$$

$$K_{JC}(T) = A + B \exp\left[C(T - T_0) \right] \text{MPa}\sqrt{m}$$

where, P_f, K_0, and T_0 are the cumulative failure probability, 63rd percentile fracture toughness value

47

and the DBTT, respectively. The temperature that provides a median fracture toughness of 100 MPa$\sqrt{m}$ is considered as the DBTT. However, the master curve equation is based on small scale yielding and neglects the influence of ductile damage. Hence, the applicability of the master curve approach is limited near the transition region and lower failure probabilities.

The temperature dependent fracture toughness K_{JC} is computed using the `AuxKernel` `K1cAux` and is output as an `AuxVariable` in the model.

```
[./K1cAux_1]
  type = K1cAux
  variable = <string>
  temperature = <string>
  transition_temperature_shift = <string>
  A = <real>
  B = <real>
  C = <real>
  T0 = <real>
  Kmin = <real> (20)
  confidence_level = <real> (0.632)
[../]
```

type	K1cAux
variable	**Required.** Name of `AuxVariable` that will hold result.
temperature	**Required.** Variable name for temperature variable. Typically `temp`.
transition_temperature_shift	**Required.** Variable name for transition temperature shift variable (given in degrees F).
A	**Required.** Parameter in the master curve function
B	**Required.** Parameter in the master curve function
C	**Required.** Parameter in the master curve function
T0	**Required.** Ductile-to-brittle transition temperature, defined as the temperature for K1C=100 MPa m$^{0.5}$ (in K).
confidence_level	The cumulative probability of fracture P_f ($0 < P_f \leq 1$).
Kmin	Lower bound fracture toughness (in MPa m$^{0.5}$).

In models with a crack, K_{JC} can be output along the crack front using the parameter `output_variables` = `'k1c_1 k1c_50'` within the `[DomainIntegral]` block.

8.4 Fracture Domain Integrals

8.4.1 J-integral

Linear elastic fracture mechanics (LEFM) is widely accepted as an appropriate methodology to assess the susceptibility of a reactor pressure vessel (RPV) to fracture during a loading event [8]. LEFM is based on the assumption that the material ahead of a crack front behaves in a linearly elastic manner, resulting in a singularity in the stress field ahead of the crack tip. The mode-I stress intensity factor, K_I is a measure of the degree of the singularity of that field on a crack under mode-I (normal to crack plane) loading.

A finite element calculation in which a crack is represented in the mesh geometry can be used to calculate the displacement and stress fields in the vicinity of the crack. One straightforward way to evaluate the stress

intensity from a finite element solution is through the J-integral [9], which provides the mechanical energy release rate. If the crack is subjected to pure mode-I loading, K_I can be calculated from J using the following relationship:

$$J = \begin{cases} \frac{1-v^2}{E} K_I^2 & \text{plane strain} \\ \frac{1}{E} K_I^2 & \text{plane stress} \end{cases}$$

where E is the Young's modulus and v is the Poisson's ratio. For assessing flaws in a RPV, plane strain conditions are assumed.

The J-integral was originally formulated as an integral over a closed curve around the crack tip in 2D. It can alternatively be expressed as an integral over a surface in 2D or a volume in 3D using the method of [10], which is more convenient for implementation within a finite element code.

Following the terminology of [11], the integrated value of the J-integral over a segment of a crack front, represented as $\bar{J}$, can be expressed as a summation of four terms:

$$\bar{J} = \bar{J}_1 + \bar{J}_2 + \bar{J}_3 + \bar{J}_4$$

where the individual terms are defined as:

$$\bar{J}_1 = \int_{V_0} \left(P_{ji} \frac{\partial u_i}{\partial X_k} \frac{\partial q_k}{\partial X_j} - W \frac{\partial q_k}{\partial X_k} \right) dV_0$$

$$\bar{J}_2 = -\int_{V_0} \left(\frac{\partial W}{\partial X_k} - P_{ji} \frac{\partial^2 u_i}{\partial X_j \partial X_k} \right) q_k dV_0$$

$$\bar{J}_3 = -\int_{V_0} \left(T \frac{\partial q_k}{\partial X_k} - \rho \frac{\partial^2 u_i}{\partial t^2} \frac{\partial u_i}{\partial X_k} q_k + \rho \frac{\partial u_i}{\partial t} \frac{\partial^2 u_i}{\partial t \partial X_k} q_k \right) dV_0$$

$$\bar{J}_4 = -\int_{A_0} t_i \frac{\partial u_i}{\partial X_k} q_k dA_0$$

In these equations, V_0 is the undeformed volume, A_0 is the combined area of the two crack faces, P_{ji} is the 1st Piola-Kirchoff stress tensor, u_i is the displacement vector, X_k is the undeformed coordinate vector, W is the stress-work density, T is the kinetic energy density, t is time, ρ is the material density, and t_i is the vector of tractions applied to the crack face.

The vector field q_k is a vector field that is oriented in the crack normal direction. This field has a magnitude of 1 between the crack tip and the inner radius of the ring over which the integral is to be performed, and drops from 1 to 0 between the inner and outer radius of that ring, and has a value of 0 elsewhere. In 3D, J is evaluated by calculating the integral $\bar{J}$ over a segment of the crack front. A separate q field is formed for each segment along the crack front, and for each ring over which the integral is to be evaluated. Each of those q fields is multiplied by a weighting function that varies from a value of 0 at the ends to a finite value in the middle of the segment. The value of J at each point on the curve is evaluated by dividing $\bar{J}$ by the integrated value of the weighting function over the segment containing that point.

The first term in $\bar{J}$, $\bar{J}_1$, represents the effects of strain energy in homogeneous material in the absence of thermal strains or inertial effects. The second term, $\bar{J}_2$, accounts for the effects of material inhomogenieties and gradients of thermal strain. For small strains, this term can be represented as:

$$\bar{J}_2 = \int_{V_0} \sigma_{ij} \left[\alpha_{ij} \frac{\partial \bar{\theta}}{\partial X_k} + \frac{\partial \alpha_{ij}}{\partial X_k} \bar{\theta} \right] q_k dV_0$$

where σ_{ij}, α_{ij} and $\bar{\theta}$ are the Cauchy stress, thermal conductivity, and difference between the current temperature and a reference temperature, respectively.

The third term in $\bar{J}$, $\bar{J}_3$, accounts for inertial effects in a dynamic analysis, and the fourth term, $\bar{J}_4$ accounts for the effects of surface tractions on the crack face.

The implementation of the J-integral calculator in Grizzly can be used for arbitrary curved 3D crack fronts, and includes the $\bar{J}_1$ and $\bar{J}_2$ terms to account for the effects of quasistatic mechanically and thermally induced strains. The last two terms have not been implemented. Because the RPV analyses are all quasistatic, there are no inertial effects, so $\bar{J}_3 = 0$. $\bar{J}_4$ would be nonzero for surface-breaking cracks because of the effects of the fluid pressure, but its contribution is expected to be negligible relative to that of the first two terms.

Pointwise values $J(s)$ are calculated from the J-integral over a crack front segment $\bar{J}$ using the expression

$$J(s) = \frac{\bar{J}(s)}{\int q(s)ds}.$$

To use the J-integral capability in Grizzly, a user specifies a set of nodes along the crack front, information used to calculate the crack normal directions along the crack front, and the inner and outer radii of the set of rings over which the integral is to be performed. Grizzly automatically defines the full set of q functions for each point along the crack front, and outputs either the value of J or K_I at each of those points. In addition, the user can ask for any other variable in the model to be output at points corresponding to those where J is evaluated along the crack front.

8.4.2 Interaction integral

The interaction integral method is based on the J-integral and makes it possible to evaluate mixed-mode stress intensity factors K_I, K_{II} and K_{III}, as well as the T-stress, in the vicinity of three-dimensional cracks. The formulation relies on superimposing Williams' solution for stress and displacement around a crack (in this context called 'auxiliary fields') and the computed finite element stress and displacement fields (called 'actual fields'). The total superimposed J can be separated into three parts: the J of the actual fields, the J of the auxiliary fields, and an interaction part containing the terms with both actual and auxiliary field quantities. The last part is called the interaction integral and for a fairly straight crack without body forces, thermal loading or crack face tractions, the interaction integral over a crack front segment can be written [12]:

$$\bar{I}(s) = \int_V \left[\sigma_{ij} u_{j,1}^{\text{aux}} + \sigma_{ij}^{\text{aux}} u_{j,1} - \sigma_{jk} \epsilon_{jk}^{\text{aux}} \delta_{1i} \right] q_{,i} \, dV$$

where σ is the stress, u is the displacement, and q is identical to the q-functions used for J-integrals. This is the formulation of the interaction integral used currently in Grizzly. It is therefore not recommended to use the interaction integral method in analyses where thermal loading, body forces or crack face tractions are important, as Grizzly does not include the necessary correction terms in the integral.

In the same way as for the J-integral, the pointwise value $I(s)$ at location s is obtained from $\bar{I}(s)$ using:

$$I(s) = \frac{\bar{I}(s)}{\int q(s)ds}.$$

Next, we relate the interaction integral $I(s)$ to stress intensity factors. By writing J^S, with actual and auxiliary fields superimposed, in terms of the mixed-mode stress intensity factors

$$J^S(s) = \frac{1-v^2}{E} \left[\left(K_I + K_I^{\text{aux}} \right)^2 + \left(K_{II} + K_{II}^{\text{aux}} \right)^2 \right] + \frac{1+v}{E} \left(K_{III} + K_{III}^{\text{aux}} \right)^2 = J(s) + J^{\text{aux}}(s) + I(s)$$

the interaction integral part evaluates to

$$I(s) = \frac{1 - v^2}{E} \left(2K_I K_I^{\text{aux}} + 2K_{II} K_{II}^{\text{aux}} \right) + \frac{1 + v}{E} \left(K_{III} K_{III}^{\text{aux}} \right)$$

To obtain individual stress intensity factors, the interaction integral is evaluated with different auxiliary fields. For instance, by choosing $K_I^{\text{aux}} = 1.0$ and $K_{II}^{\text{aux}} = K_{III}^{\text{aux}} = 0$ and computing the volume integral $I(s)$ over the actual and auxiliary fields, K_I can be solved for.

If all three interaction integral stress intensity factors are computed, Grizzly includes an option to output an equivalent stress intensity factor computed by:

$$K_{eq} = \sqrt{K_I^2 + K_{II}^2 + \frac{K_{III}^2}{1 - v}}.$$

The T-stress is the first second-order parameter in Williams' expansion of stress at a crack tip and is a constant stress parallel to the crack [13], [14]. Contrary to J, T-stress depends on geometry and size and can give a more accurate description of the stresses and strains around a crack tip than J alone. The T-stress characterizes the crack-tip constraint and a negative T-stress is associated with loss of constraint and a higher fracture toughness than would be predicted from a one-parameter J description of the load on the crack. T-stresses can be calculated with the interaction integral methodology using appropriate auxiliary fields [15]. The current implementation of the T-stress computation is valid for two-dimensional and three-dimensional cracks under Mode I loading.

8.4.3 Usage

To use the J-integral or interaction integral capability, a `DomainIntegral` block is needed in the input file. The domain integrals operate on a set of crack front nodes (in standard FEM) or points (in XFEM). In the former, the crack front should be defined as a nodeset in the mesh file and is supplied in the parameter boundary.

```
[DomainIntegral]
  integrals = <string list>
  boundary = <string list>
  crack_direction_method = <string>
  crack_direction_vector = <vector>
  crack_mouth_boundary = <string list>
  2d = <bool> (false)
  axis_2d = <0, 1, or 2 for x, y, or z>
  radius_inner = <real list>
  radius_outer = <real list>
  symmetry_plane = <0, 1, or 2 for x, y, or z>
  output_variable = <string list>
  convert_J_to_K = <bool> (false)
  position_type = <Distance or Angle> (Distance)
  q_function_type = <Geometry or Topology> (Geometry)
[]
```

`integrals`	**Required.** List of quantities to compute. Options are JIntegral, InteractionIntegralKI, InteractionIntegralKII, InteractionIntegralKIII, and InteractionIntegralT.

`boundary`	The boundary id(s) for the crack front node nodeset. Must use either this or `crack_front_points`.
`crack_front_points`	Set of points to define a crack front. Must use either this or `boundary`.
`crack_direction_method`	**Required.** Method to determine direction of crack propagation. One of CrackDirectionVector, CrackMouth, or CurvedCrackFront.
`crack_direction_vector`	Direction of crack propagation. Required if `crack_direction_method` is CrackDirectionVector.
`crack_mouth_boundary`	Boundaries whose average coordinate defines the crack mouth. Required if `crack_direction_method` is CrackMouth.
`crack_end_direction_method`	Method to determine direction of crack propagation at ends of crack. One of NoSpecialTreatment and CrackDirectionVector.
`crack_direction_vector_end_1`	Direction of crack propagation for the node at end 1 of the crack. Required if `crack_end_direction_method` is CrackDirectionVector.
`crack_direction_vector_end_2`	Direction of crack propagation for the node at end 2 of the crack. Required if `crack_end_direction_method` is CrackDirectionVector.
`intersecting_boundary`	Boundaries intersected by ends of crack.
`q_function_type`	The method used to define the integration domain. One of Geometry or Topology.
`radius_inner`	Inner radii for volume integral domains. Used if `q_function_type` is Geometry.
`radius_outer`	Outer radii for volume integral domains. Used if `q_function_type` is Geometry.
`ring_first`	The first ring of elements for volume integral domain. Used if `q_function_type` is Topology.
`ring_last`	The last ring of elements for volume integral domain. Used if `q_function_type` is Topology.
`2d`	Treat body as two-dimensional.
`axis_2d`	Out of plane axis for models treated as two-dimensional (0=x, 1=y, 2=z).
`symmetry_plane`	Account for a symmetry plane passing through the plane of the crack, normal to the specified axis (0=x, 1=y, 2=z)
`output_variable`	Variable values to be reported along the crack front.
`convert_J_to_K`	Convert J-integral to stress intensity factor K.
`position_type`	The method used to calculate position along crack front. One of Angle or Distance. If Angle, must have a `crack_mouth_boundary`.

The following parameters apply only to the computation of interaction integrals:

`poissons_ratio`	**Required.** Poisson's ratio.
`youngs_modulus`	**Required.** Young's modulus.
`block`	**Required.** Block ids where InteractionIntegralAuxFields is defined.

`equivalent_k`	Calculate an equivalent stress intensity factor K_{eq} from K_I, K_{II} and K_{III}, assuming self-similar crack growth.
`disp_x`	**Required.** Variable name for displacement variable in x direction. Typically `disp_x`.
`disp_y`	**Required.** Variable name for displacement variable in y direction. Typically `disp_y`.
`disp_z`	**Required.** Variable name for displacement variable in z direction. Typically `disp_z`.

The parameter `2d` determines if the crack front will be treated as two- or three-dimensional. If the crack is treated as three-dimensional, domain integral results will be output for each node or point along the crack front. If the crack is treated as two-dimensional, whether the mesh is two- or three-dimensional, it is assumed to be a straight crack and only one value for the entire crack front is output. In this case, the crack front nodes or points should form a line parallel to the coordinate axis given in `axis_2d`. Note that if the crack is treated as three-dimensional the domain integral results will be output as comma separated files using VectorPostprocessors. To turn on this output, `csv` must be set to `true` in the Outputs block.

The integration domain around the crack tip is determined by the `q_function_type` and a combination of either `radius_inner` and `radius_outer` or `ring_first` and `ring_last`. The default method to create the *q* function is by defining inner and outer radii relative to the crack tip of the integration domains. This is the geometry-based method, which works for sharp and blunt crack tips. The mesh topology-based *q* function type creates rings of elements around the crack tip and currently only works for sharp crack tips.

At free surfaces or symmetry planes, the automatic computation of the direction of crack extension can be prone to errors. If `crack_end_direction_method` is set to CrackDirectionVector, the crack directions at the two crack ends are provided using the parameters `crack_direction_vector_end_1` and `crack_direction_vector_end_2`. If there are spurious results at the node next to an end node, this can be helped by supplying the indices of the boundaries intersected by the end nodes as a list to the parameter `intersecting_boundary`.

9 Demonstration RPV Analysis

Demonstration models of a PWR RPV subjected to a transient loading scenario are included in the Grizzly code repository under the `demonstration` directory. These include a full 3D model of the RPV and a small 2D axisymmetric model of a slice of the RPV. Examples of transfers of results from both of these models to a local fracture analysis of an axis-aligned flaw are also included.

9.1 Global RPV Model (2D Axisymmetric)

Because the region of the RPV that experiences the highest irradiation dose is in the beltline of the vessel, the response of the vessel in that region is typically of the most interest. The thermo-mechanical response of that region of the vessel can be characterized as that of an infinite cylinder. A simple 2D model of a single strip of elements, with appropriate boundary conditions, can be used to represent this response. This provides an accurate simulation of the response of this region of the vessel with minimal computational cost.

The 2D strip model of the RPV is in the directory `demonstration/rpv_global/2d_strip`. This directory includes the journal file to generate the mesh with Cubit, the mesh, and a Grizzly input file. The Grizzly input file for this model is named `2d_strip.i`.

This model reads in time histories of the heat transfer coefficient, coolant pressure, and coolant temperature from files in the `demonstration/rpv_global/loadhist` directory.

This model has fixed displacement boundary conditions on the bottom surface of the strip in the y direction, and uses an `EqualValueBoundaryConstraint` block to constrain the displacement in the y direction to be equal across the top surface of the model. This represents conditions for an infinite cylinder that is free to expand in its axial direction, with planar sections remaining planar.

Figure 5: Mesh used for 2D axisymmetric strip RPV model.

9.2 Global RPV Model (3D)

The 3D global model of the RPV is in the directory `demonstration/rpv_global/3d`. This directory includes the journal file to generate the mesh with Cubit, the mesh, and a Grizzly input file. The Grizzly input file for this model is named `3d_rpv.i`. This model is representative of geometry typical of a PWR RPV, and is based entirely on openly-available data. It is important to note that this model does not represent an actual RPV.

This model reads in time histories of the heat transfer coefficient, coolant pressure, and coolant temperature from files in the `demonstration/rpv_global/loadhist` directory.

This model is quite large, and should be run in parallel.

9.3 Local Fracture Submodels

The directory `demonstration/rpv_fracture_submodel` contains multiple variants of submodels for local fracture analysis of flaws. There are two versions of the geometry modeled in this directory: an axial flaw and a circumferential flaw. The `mesh` directory contains the Cubit journal files and Exodus meshes for both of these flaw orientations.
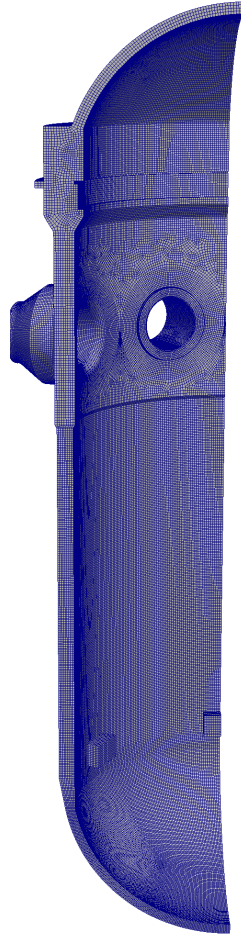
Figure 6: Mesh used for 3D RPV model.

The `transfer_from_2d_strip` directory contains input files for both of these models using the results from the 2D strip model to drive the boundary conditions. These use `Axisymmetric2D3DSolutionFunction` to map the displacement and temperature histories from the 2D strip global model.

The `transfer_from_3d` directory contains input files for both of these models using the results from the 2D strip model to drive the boundary conditions. These use `SolutionFunction` to map the displacement and temperature histories from the 3D strip global model.

Aside from the differences in the sources of the displacement and temperature fields, these two sets of models are identical. Both of these use `DomainIntegral` to compute fracture domain integrals for points along the crack front.

As all of these submodels read data from the output of global models, the corresponding global models must be run first before these models are run.
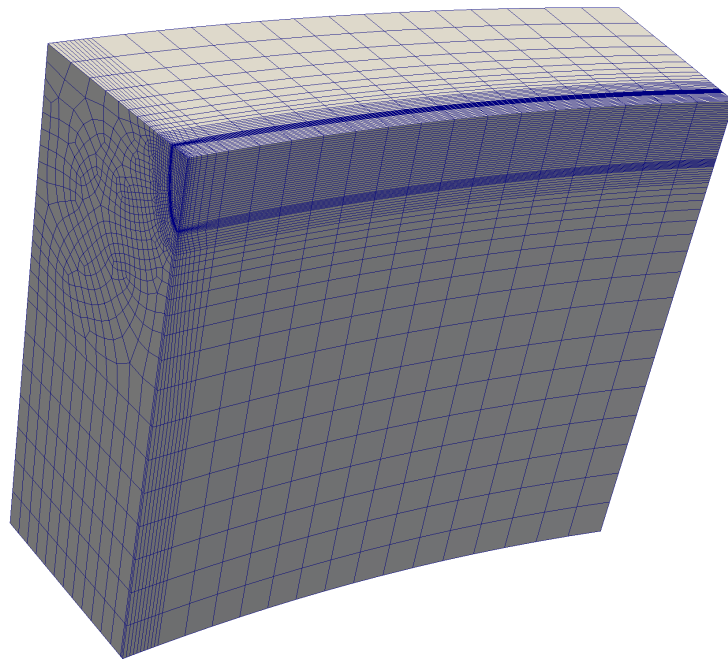
Figure 7: Mesh used for local fracture model of axial surface-breaking flaw.

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